ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{\tan^5\frac{A}{2}}{\sin^3 A}+\frac{\tan^5\frac{B}{2}}{\sin^3 B}+\frac{\tan^5\frac{C}{2}}{\sin^3 C}\geq\frac{8}{27}$$

Proposed by Zaza Mzvavanadze-Georgia

Solution by Tapas Das-India

$$\sum\sin A=\frac{s}{R}\overset{Mitrinovic}{\leq}\frac{3\sqrt{3}}{2}\ (1)\ and\ \frac{4R+r}{s}\overset{Doucet}{\geq}\sqrt{3}\ (2)$$

$$:\left(\frac{\tan^5\frac{A}{2}}{\sin^3 A}+\frac{\tan^5\frac{B}{2}}{\sin^3 B}+\frac{\tan^5\frac{C}{2}}{\sin^3 C}\right)\left(\sum\sin A\right)^3(1+1+1)\overset{Holder}{\geq}\left(\sum\tan\frac{A}{2}\right)^5$$

$$or\left(\frac{\tan^5\frac{A}{2}}{\sin^3 A}+\frac{\tan^5\frac{B}{2}}{\sin^3 B}+\frac{\tan^5\frac{C}{2}}{\sin^3 C}\right)\geq\frac{\left(\sum\tan\frac{A}{2}\right)^5}{(\sum\sin A)^3(1+1+1)}=\frac{\left(\frac{4R+r}{s}\right)^5}{\left(3\left(\frac{s}{R}\right)^3\right)}\overset{(1)\&(2)}{\geq}$$

$$\geq\frac{3^{\frac{5}{2}}}{\frac{243\sqrt{3}}{8}}=\frac{9\cdot 8}{243}=\frac{8}{27}$$

$$Equality\ for\ A=B=C$$