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In ΔABC the following relationship holds:

$$\frac{n_a g_a}{sinA}+\frac{n_b g_b}{sinB}+\frac{n_c g_c}{sinC}\geq\ 18\sqrt{3}r^2$$

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Solution by Daniel Sitaru-Romania

$$\frac{n_a g_a}{sinA}+\frac{n_b g_b}{sinB}+\frac{n_c g_c}{sinC}=\sum_{cyc}\frac{h_a h_a}{sinA}=\sum_{cyc}\frac{\frac{2F}{a}\cdot\frac{2F}{a}}{sinA}=$$

$$=4F^2\cdot\sum_{cyc}\frac{1}{a^2 sinA}=4F^2\cdot\sum_{cyc}\frac{1}{a^2\cdot\frac{a}{2R}}=8RF^2\sum_{cyc}\frac{1^4}{a^3}\overset{RADON}{\geq}$$

$$\geq\ 8RF^2\cdot\frac{(1+1+1)^4}{(a+b+c)^3}=8Rr^2s^2\cdot\frac{81}{8s^3}=\frac{81Rr^2}{s}\overset{MITRINOVIC}{\geq}$$

$$\geq\frac{81Rr^2}{\frac{3\sqrt{3}R}{2}}=\frac{54r^2}{\sqrt{3}}=18\sqrt{3}r^2$$

Equality holds for $a=b=c$.