ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a^n}{\cos^2\frac{A}{2}}+\frac{b^n}{\cos^2\frac{B}{2}}+\frac{c^n}{\cos^2\frac{C}{2}}\geq 2^{n+2}\cdot 3^{\frac{n}{2}}\cdot r^n, n\in N$$

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Solution by Tapas Das-India

$$WLOG\ a\geq b\geq c\ then\ a^n\geq b^n\geq c^n\ and$$

$$\cos\frac{A}{2}\leq\cos\frac{B}{2}\leq\cos\frac{C}{2},\sum\cos^2\frac{A}{2}=2+\frac{r}{2R}\overset{Euler}{\leq}\frac{9}{4}$$

$$\frac{a^n}{\cos^2\frac{A}{2}}+\frac{b^n}{\cos^2\frac{B}{2}}+\frac{c^n}{\cos^2\frac{C}{2}}\overset{Chebyshev}{\geq}$$

$$\frac{1}{3}\left(\sum a^n\right)\left(\sum\frac{1}{\cos^2\frac{A}{2}}\right)\geq\frac{1}{3.3^{n-1}}(a+b+c)^n\frac{9}{\sum\cos^2\frac{A}{2}}(CBS)\geq$$

$$\geq\frac{2^n s^n.9}{3.3^{n-1}\frac{9}{4}}\geq 2^{n+2}r^n 3^{\frac{3n}{2}}.3^{-n}=2^{n+2}3^{\frac{n}{2}}r^n$$

$$Equality\ for\ a=b=c$$