ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{e^a\left(e^{\tan\frac{B}{2}}+e^{\tan\frac{C}{2}}\right)}{e^b+e^c}+\frac{e^b\left(e^{\tan\frac{C}{2}}+e^{\tan\frac{A}{2}}\right)}{e^c+e^a}+\frac{e^c\left(e^{\tan\frac{A}{2}}+e^{\tan\frac{B}{2}}\right)}{e^a+e^b}\geq 3\,e^{\frac{\sqrt{3}}{3}}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\textit{Walter Janous' inequality}:$$
$$\textit{Let } a,b,c \textit{ and } x,y,z \textit{ be positive real number then}:$$

$$\frac{x}{y+z}(b+c)+\frac{y}{z+x}(c+a)+\frac{z}{x+y}(a+b)\geq\sqrt{3(ab+bc+ca)}\quad(1)$$

$$e^{2\sum\tan\frac{A}{2}}=e^{\frac{2(4R+r)}{s}}\overset{Doucet}{\geq}e^{2\sqrt{3}}\quad(2)$$

$$\frac{e^a\left(e^{\tan\frac{B}{2}}+e^{\tan\frac{C}{2}}\right)}{e^b+e^c}+\frac{e^b\left(e^{\tan\frac{C}{2}}+e^{\tan\frac{A}{2}}\right)}{e^c+e^a}+\frac{e^c\left(e^{\tan\frac{A}{2}}+e^{\tan\frac{B}{2}}\right)}{e^a+e^b}=$$

$$=\sum\frac{e^a}{e^b+e^c}\left(e^{\tan\frac{B}{2}}+e^{\tan\frac{C}{2}}\right)\overset{(1)}{\geq}\sqrt{3\left(e^{\tan\frac{A}{2}}.e^{\tan\frac{B}{2}}+e^{\tan\frac{B}{2}}.e^{\tan\frac{C}{2}}+e^{\tan\frac{A}{2}}.e^{\tan\frac{C}{2}}\right)}\overset{AM-GM\&(2)}{\geq}$$

$$\geq\sqrt{9\left(e^{2\sqrt{3}}\right)^{\frac{1}{3}}}=3\,e^{\frac{\sqrt{3}}{3}}$$

$$\textit{Equality for } a=b=c$$