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In ΔABC the following relationship holds:

$$\frac{h_a^n}{\sin\frac{A}{2}\sin\frac{B}{2}}+\frac{h_b^n}{\sin\frac{B}{2}\sin\frac{C}{2}}+\frac{h_c^n}{\sin\frac{C}{2}\sin\frac{A}{2}}\ge 4\cdot 3^{n+1}\cdot r^n \qquad n\in N$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\prod\sin\frac{A}{2}=\frac{r}{4R}\overset{Euler}{\le}\frac{1}{8}\ (1)$$

$$\sqrt[3]{h_ah_bh_c}\overset{Gm-Hm}{\ge}\frac{3}{\sum\frac{1}{h_a}}=3r\ \ (2)$$

$$\frac{h_a^n}{\sin\frac{A}{2}\sin\frac{B}{2}}+\frac{h_b^n}{\sin\frac{B}{2}\sin\frac{C}{2}}+\frac{h_c^n}{\sin\frac{C}{2}\sin\frac{A}{2}}\overset{Am-Gm}{\ge}$$

$$\ge\frac{3(h_ah_bh_c)^{\frac{n}{3}}}{\left(\prod\sin\frac{A}{2}\right)^{\frac{2}{3}}}\overset{(1)\&(2)}{\ge}3.3^nr^n.4=4.3^{n+1}\cdot r^n$$

$$Equality\ holds\ for\ a=b=c$$