ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\pi^{3\tan^2\frac{A}{2}}\left(\csc\frac{B}{2}+\csc\frac{C}{2}\right)+\pi^{3\tan^2\frac{B}{2}}\left(\csc\frac{C}{2}+\csc\frac{A}{2}\right)+\pi^{3\tan^2\frac{C}{2}}\left(\csc\frac{A}{2}+\csc\frac{B}{2}\right)\geq 12\pi$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\prod\csc\frac{A}{2}=\frac{4R}{r}\overset{Euler}{\geq}8\ (1)\ and$$

$$\sum 3\tan^2\frac{A}{2}\overset{CBS}{\geq}\geq\left(\sum\tan\frac{A}{2}\right)^2=\left(\frac{4R+r}{s}\right)^2\overset{Doucet}{\geq}3\ (2)$$

$$\pi^{3\tan^2\frac{A}{2}}\left(\csc\frac{B}{2}+\csc\frac{C}{2}\right)+\pi^{3\tan^2\frac{B}{2}}\left(\csc\frac{C}{2}+\csc\frac{A}{2}\right)+\pi^{3\tan^2\frac{C}{2}}\left(\csc\frac{A}{2}+\csc\frac{B}{2}\right)=$$

$$=\sum\pi^{3\tan^2\frac{A}{2}}\left(\csc\frac{B}{2}+\csc\frac{C}{2}\right)\overset{AM-GM}{\geq}$$

$$\geq 2\sum\pi^{3\tan^2\frac{A}{2}}\sqrt{\csc\frac{A}{2}\csc\frac{B}{2}}\overset{AM-GM}{\geq}6\left(\pi^{\sum 3\tan^2\frac{A}{2}}\prod\csc\frac{A}{2}\right)^{\frac{1}{3}}\overset{(1)\&(2)}{\geq}6(\pi^3.8)^{\frac{1}{3}}=12\pi$$

$$Equality\ holds\ for\ A=B=C.$$