ROMANIAN MATHEMATICAL MAGAZINE

**In $\Delta ABC$ the following relationship holds:**

$$e^{3\tan^2\frac{A}{2}}\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)+e^{3\tan^2\frac{B}{2}}\left(\tan\frac{C}{2}+\tan\frac{A}{2}\right)+e^{3\tan^2\frac{C}{2}}\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)\geq 2\sqrt{3}\,e$$

*Proposed by Zaza Mzhavanadze-Georgia*

*Solution by Tapas Das-India*

$$\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)\left(\tan\frac{C}{2}+\tan\frac{A}{2}\right)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)$$
$$=\left(\sum\tan\frac{A}{2}\right)\left(\sum\tan\frac{A}{2}\tan\frac{B}{2}\right)-\prod\tan\frac{A}{2}=\frac{4R+r}{s}-\frac{r}{s}=\frac{4R}{s}\overset{Mitrinovic}{\geq}$$
$$\geq\frac{4}{s}\cdot\frac{2s}{3\sqrt{3}}=\left(\frac{2}{\sqrt{3}}\right)^3\quad(1)$$

$$\sum 3\tan^2\frac{A}{2}\overset{CBS}{\geq}\left(\sum\tan\frac{A}{2}\right)^2=\left(\frac{4R+r}{s}\right)^2\overset{Doucet}{\geq}3\ (2)$$

$$e^{3\tan^2\frac{A}{2}}\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)+e^{3\tan^2\frac{B}{2}}\left(\tan\frac{C}{2}+\tan\frac{A}{2}\right)+e^{3\tan^2\frac{C}{2}}\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)\geq$$

$$\overset{AM-GM}{\geq}3\left(e^{\sum 3\tan^2\frac{A}{2}}\left(\tan\frac{B}{2}+\tan\frac{C}{2}\right)\left(\tan\frac{C}{2}+\tan\frac{A}{2}\right)\left(\tan\frac{A}{2}+\tan\frac{B}{2}\right)\right)^{\frac{1}{3}}\overset{(1)\&(2)}{\geq}$$
$$\geq 3\left(e^3\left(\frac{2}{\sqrt{3}}\right)^3\right)^{\frac{1}{3}}=3.e.\frac{2}{\sqrt{3}}=2\sqrt{3}\,e$$

*Equality holds for* $A=B=C$