ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{n_a+g_b}{r_c}+\frac{n_b+g_c}{r_a}+\frac{n_c+g_a}{r_b}\geq\frac{12r}{R}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$h_a=\frac{bc}{2R},h_b=\frac{ac}{2R},h_c=\frac{ab}{2R}\ and\ r_ar_br_c=s^2r,abc=4Rrs$$

$$\frac{n_a+g_b}{r_c}+\frac{n_b+g_c}{r_a}+\frac{n_c+g_a}{r_b}\geq\frac{h_a+h_b}{r_c}+\frac{h_b+h_c}{r_a}+\frac{h_c+h_a}{r_b}=$$

$$=\frac{1}{2R}\left(\frac{bc+ac}{r_c}+\frac{ab+ac}{r_a}+\frac{bc+ba}{r_b}\right)\overset{AM-GM}{\geq}\frac{1}{2R}\left(\frac{2\sqrt{c^2ab}}{r_c}+\frac{2\sqrt{a^2bc}}{r_a}+\frac{2\sqrt{b^2ac}}{r_b}\right)=$$

$$=\frac{1}{R}\left(\frac{\sqrt{c^2ab}}{r_c}+\frac{\sqrt{a^2bc}}{r_a}+\frac{\sqrt{b^2ac}}{r_b}\right)\overset{AM-GM}{\geq}\frac{3}{R}.\left(\frac{\sqrt{c^2ab}}{r_c}.\frac{\sqrt{a^2bc}}{r_a}.\frac{\sqrt{b^2ac}}{r_b}\right)^{\frac{1}{3}}=$$

$$=\frac{3}{R}\left(\frac{a^2b^2c^2}{r_ar_br_c}\right)^{\frac{1}{3}}=\frac{3}{R}\left(\frac{16R^2r^2s^2}{s^2r}\right)^{\frac{1}{3}}\overset{Euler}{\geq}\frac{3}{R}\left(\frac{16(2r)^2r^2}{r}\right)^{\frac{1}{3}}=\frac{3}{R}(64r^3)^{\frac{1}{3}}=\frac{3}{R}(4r)=\frac{12r}{R}$$

$$Equality\ for\ a=b=c$$