ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{(a+b)^3}{m_a+r_b}+\frac{(b+c)^3}{m_b+r_c}+\frac{(c+a)^3}{m_c+r_a}\geq\frac{192\sqrt{3}r^3}{R}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\frac{(a+b)^3}{m_a+r_b}+\frac{(b+c)^3}{m_b+r_c}+\frac{(c+a)^3}{m_c+r_a}\overset{Holder}{\geq}\frac{(2a+2b+2c)^3}{3(\sum m_a+\sum r_a)}\overset{Leuenberger}{\geq}$$

$$\geq\frac{64s^3}{3(4R+r).2}\overset{Mitrinovic\,\&\,Euler}{\geq}64\cdot 27r^2\cdot\frac{3\sqrt{3}r}{6\frac{9R}{2}}=\frac{192\sqrt{3}r^3}{R}$$

$$Equality\ holds\ \ for\ a=b=c$$