ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{(a+b)^3}{\sin^2 A}+\frac{(b+c)^3}{\sin^2 B}+\frac{(c+a)^3}{\sin^2 C}\geq 4\cdot\left(4\sqrt{3}r\right)^3$$

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Solution by Daniel Sitaru-Romania

$$\frac{(a+b)^3}{\sin^2 A}+\frac{(b+c)^3}{\sin^2 B}+\frac{(c+a)^3}{\sin^2 C}=\sum_{cyc}\frac{(a+b)^3}{\sin^2 A}=$$

$$=\sum_{cyc}\frac{(a+b)^3}{\left(\frac{a}{2R}\right)^2}=4R^2\sum_{cyc}\frac{(a+b)^3}{a^2}\overset{RADON}{\geq}$$

$$\geq 4R^2\cdot\frac{(a+b+b+c+c+a)^3}{(a+b+c)^2}=4R^2\cdot 8\cdot\frac{(a+b+c)^3}{(a+b+c)^2}=4R^2\cdot 8\cdot 2s\geq$$

$$\overset{EULER}{\geq} 4\cdot 4r^2\cdot 16\cdot s\overset{MITRINOVIC}{\geq} 4\cdot 4^3r^2\cdot 3\sqrt{3}r=4\cdot\left(4\sqrt{3}r\right)^3$$

Equality holds for $a=b=c$.