ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{(a^2+b^2)^2}{\sin^2 A}+\frac{(b^2+c^2)^2}{\sin^2 B}+\frac{(c^2+a^2)^2}{\sin^2 C}\geq 9\cdot(4r)^4$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\frac{(a^2+b^2)^2}{\sin^2 A}+\frac{(b^2+c^2)^2}{\sin^2 B}+\frac{(c^2+a^2)^2}{\sin^2 C}=\sum\frac{(a^2+b^2)^2}{\sin^2 A}=$$

$$=4R^2\sum\frac{(a^2+b^2)^2}{a^2}\overset{Bergstrom}{\geq}\frac{4R^2(2a^2+2b^2+2c^2)^2}{a^2+b^2+c^2}=$$

$$=16R^2(a^2+b^2+c^2)\overset{Neuberg}{\geq}16R^2.36r^2\overset{Euler}{\geq}16(2r)^2 36r^2=9(4r)^4$$

$$Equality\ holds\ for\ a=b=c$$