If in $\Delta ABC, abc = 1$ then:

$$\sum \frac{b^2 \cot\frac{C}{2} + c^2 \cot\frac{B}{2}}{b+c} \geq 3\sqrt{3}$$

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$$WLOG\ a \geq b \geq c\ then \cot\frac{A}{2} \leq \cot\frac{B}{2} \leq \cot\frac{C}{2}$$

$$\frac{b^2 \cot\frac{C}{2} + c^2 \cot\frac{B}{2}}{b+c} \overset{Chebyshev}{\geq} \frac{\frac{1}{2}(b^2+c^2)\left(\cot\frac{C}{2} + \cot\frac{B}{2}\right)}{b+c} \overset{CBS}{\geq}$$

$$\geq \frac{\frac{1}{2}\frac{(b+c)^2}{2}\left(\cot\frac{C}{2} + \cot\frac{B}{2}\right)}{b+c} = \frac{\frac{(b+c)^2}{4}\left(\cot\frac{C}{2} + \cot\frac{B}{2}\right)}{b+c} =$$

$$= \frac{b+c}{4}\left(\cot\frac{C}{2} + \cot\frac{B}{2}\right) \overset{AM-GM}{\geq} \sqrt{bc}\sqrt{\cot\frac{B}{2}\cot\frac{C}{2}} \quad (1)$$

$$\sum \frac{b^2 \cot\frac{C}{2} + c^2 \cot\frac{B}{2}}{b+c} \overset{(1)}{\geq} \sum \sqrt{bc}\sqrt{\cot\frac{B}{2}\cot\frac{C}{2}} \overset{AM-GM}{\geq}$$

$$\geq 3\left(abc\prod \cos\frac{A}{2}\right)^{\frac{1}{3}} \overset{abc=1}{=} 3\left(\frac{s}{r}\right)^{\frac{1}{3}} \overset{Mitrinovic}{\geq} 3\sqrt{3}$$

Equality holds for $a = b = c = 1$.