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In ΔABC the following relationship holds:

$$\sum \frac{(a+b)^4}{\sin\frac{A}{2}\left(1+\sin\frac{B}{2}\right)} \ge 36(4r)^4$$

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Solution by Tapas Das-India

$$\sum \sin\frac{A}{2} \overset{Jensen}{\le} 3\sin\frac{\pi}{6} = \frac{3}{2}\ (1)$$

$$\left(\sum \frac{(a+b)^4}{\sin\frac{A}{2}\left(1+\sin\frac{B}{2}\right)}\right)\left(\sum \sin\frac{A}{2}\right)\left(\sum 1+\sin\frac{B}{2}\right)(1+1+1) \overset{Holder}{\ge} (2a+2b+2c)^4$$

$$\sum \frac{(a+b)^4}{\sin\frac{A}{2}\left(1+\sin\frac{B}{2}\right)} \ge \frac{16(2s)^4}{\left(\sum \sin\frac{A}{2}\right)\left(\sum 1+\sin\frac{B}{2}\right)(1+1+1)} \overset{(1)\&Mitrinovic}{\ge}$$

$$\ge \frac{256\left(3\sqrt{3}r\right)^4}{\frac{3}{2}\left(3+\frac{3}{2}\right).3} = \frac{(4.9)(81)(4r)^4}{81} = 36(4r)^4$$

Equality holds iff ΔABC is equilateral