ROMANIAN MATHEMATICAL MAGAZINE

If $n \in N$ then in ΔABC the following relationship holds:

$$\frac{a^n}{\sin^2 A}+\frac{b^n}{\sin^2 B}+\frac{c^n}{\sin^2 C} \ge 2^{n+2}\cdot 3^{\frac{n}{2}}\cdot r^n$$

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$$\frac{a^n}{\sin^2 A}+\frac{b^n}{\sin^2 B}+\frac{c^n}{\sin^2 C} \overset{A-G}{\ge} 3\sqrt[3]{\frac{(abc)^n}{(\sin A\,.\sin B\,.\,\sin C)^2}} \ge$$

$$abc = 4RS \overset{Euler\ \ Mitrinovic}{\ge} 2^3\,.\,3^{\frac{3}{2}}\,.\,r^3$$

$$\boxed{(abc)^n \ge 2^{3n}\,.\,3^{\frac{3n}{2}}\,.\,r^{3n}}\ (1)$$

$$\overset{(1)}{\ge} 3\,.\,2^n\,.\,3^{\frac{n}{2}}\,.\,r^n\,.\sqrt[3]{\frac{1}{(\sin A\,.\,\sin B\,.\,\sin C)^2}} \overset{(4)}{\ge}$$

$$\boxed{\sin A\,.\,\sin B\,.\,\sin C = \frac{S}{2R^2}}\ (2)\quad \boxed{S \le \frac{3\sqrt{3}R^2}{4}}\ (3)$$

From (2) *and* (3) *we have* $\boxed{\sin A\,.\,\sin B\,.\,\sin C \le \frac{3\sqrt{3}}{8}}$ (4)

$$\overset{4}{\ge} 3\,.\,2^n\,.\,3^{\frac{n}{2}}\,.\,r^n\,.\sqrt[3]{\left(\frac{8}{3\sqrt{3}}\right)^2} = 2^{n+2}\,.\,3^{\frac{n}{2}}\,.\,r^n$$