ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{m_a}{a}+\frac{w_b}{b}+\frac{h_c}{c}\geq\frac{3\sqrt{3}r}{R}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Daniel Sitaru-Romania

$$\frac{m_a}{a}+\frac{w_b}{b}+\frac{h_c}{c}\geq\frac{h_a}{a}+\frac{h_b}{b}+\frac{h_c}{c}=\frac{ah_a}{a^2}+\frac{bh_b}{b^2}+\frac{ch_c}{c^2}=$$

$$=\frac{2F}{a^2}+\frac{2F}{b^2}+\frac{2F}{c^2}=2F\left(\frac{1}{a^2}+\frac{1}{b^2}+\frac{1}{c^2}\right)=2F\left(\frac{1^3}{a^2}+\frac{1^3}{b^2}+\frac{1^3}{c^2}\right)\geq$$

$$\overset{RADON}{\overbrace{\geq}}2F\cdot\frac{(1+1+1)^3}{(a+b+c)^2}=\frac{54F}{4s^2}=\frac{27rs}{2s^2}=\frac{27r}{2s}\geq$$

$$\overset{MITRINOVIC}{\overbrace{\geq}}\frac{27r}{2\cdot\frac{3\sqrt{3}R}{2}}=\frac{9r}{\sqrt{3}R}=\frac{3\sqrt{3}r}{R}$$

Equality holds for $a=b=c$.