ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{\cot^n\frac{A}{2}}{a}+\frac{\cot^n\frac{B}{2}}{b}+\frac{\cot^n\frac{C}{2}}{c}\geq\frac{3^{\frac{n+1}{2}}}{R}\ ,n\in N$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$abc=4Rrs\overset{Euler\ \&\ Mitrinovic}{\leq}4R\frac{R}{2}\frac{3\sqrt{3}R}{2}=3\sqrt{3}R^3\ and$$

$$\prod\cot\frac{A}{2}=\frac{s}{r}\overset{Mitrinovic}{\geq}3\sqrt{3}=3^{\frac{3}{2}}$$

$$\frac{\cot^n\frac{A}{2}}{a}+\frac{\cot^n\frac{B}{2}}{b}+\frac{\cot^n\frac{C}{2}}{c}\overset{AM-GM}{\geq}3\sqrt[3]{\frac{\left(\prod\cot\frac{A}{2}\right)^n}{abc}}\geq3\sqrt[3]{\frac{3^{\frac{3n}{2}}}{R3^{\frac{3}{2}}}}=\frac{3.3^{\frac{n-1}{2}}}{R}=\frac{3^{\frac{n+1}{2}}}{R}$$

$$Equality\ holds\ for\ a=b=c.$$