ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC, the following relationship holds :

$$\frac{m_a^5}{w_b^5(w_b+h_c)^2}+\frac{w_b^5}{h_c^5(h_c+m_a)^2}+\frac{h_c^5}{m_a^5(m_a+w_b)^2}\geq\frac{1}{3R^2}\left(\frac{2r}{R}\right)^{15}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\frac{m_a^5}{w_b^5(w_b+h_c)^2}+\frac{w_b^5}{h_c^5(h_c+m_a)^2}+\frac{h_c^5}{m_a^5(m_a+w_b)^2}$$

$$\geq\frac{h_a^5}{m_b^5(m_b+m_c)^2}+\frac{h_b^5}{m_c^5(m_c+m_a)^2}+\frac{h_c^5}{m_a^5(m_a+m_b)^2}$$

$$\overset{\text{Panaitopol}}{\geq}\frac{\left(\frac{2rs}{a}\right)^5}{\left(\frac{Rs}{b}\right)^5(m_b+m_c)^2}+\frac{\left(\frac{2rs}{b}\right)^5}{\left(\frac{Rs}{c}\right)^5(m_c+m_a)^2}+\frac{\left(\frac{2rs}{c}\right)^5}{\left(\frac{Rs}{a}\right)^5(m_a+m_b)^2}$$

$$=\left(\frac{2r}{R}\right)^5\cdot\left(\frac{\left(\frac{b}{a}\right)^5}{(m_b+m_c)^2}+\frac{\left(\frac{c}{b}\right)^5}{(m_c+m_a)^2}+\frac{\left(\frac{a}{c}\right)^5}{(m_a+m_b)^2}\right)\overset{\text{Holder}}{\geq}$$

$$\left(\frac{2r}{R}\right)^5\cdot\frac{\left(\sum_{cyc}\frac{b}{a}\right)^5}{27\sum_{cyc}(m_b+m_c)^2}\overset{A-G}{\geq}\left(\frac{2r}{R}\right)^5\cdot\frac{243}{27.2\sum_{cyc}(m_b^2+m_c^2)}=\left(\frac{2r}{R}\right)^5\cdot\frac{9}{4\sum_{cyc}m_a^2}$$

$$=\left(\frac{2r}{R}\right)^5\cdot\frac{9}{4.\frac{3}{4}\sum_{cyc}a^2}\overset{\text{Leibnitz}}{\geq}\left(\frac{2r}{R}\right)^5\cdot\frac{9}{3.9R^2}=\frac{1}{3R^2}\frac{\left(\frac{2r}{R}\right)^{15}}{\left(\frac{2r}{R}\right)^{10}}\overset{\text{Euler}}{\geq}\frac{1}{3R^2}\frac{\left(\frac{2r}{R}\right)^{15}}{(1)^{10}}$$

$$\therefore\frac{m_a^5}{w_b^5(w_b+h_c)^2}+\frac{w_b^5}{h_c^5(h_c+m_a)^2}+\frac{h_c^5}{m_a^5(m_a+w_b)^2}\geq\frac{1}{3R^2}\left(\frac{2r}{R}\right)^{15}$$

$\forall\,\Delta$ ABC,'' ='' iff Δ ABC is equilateral (QED)