ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC, the following relationship holds :

$$\frac{m_a^2+m_b^2}{(w_a+w_b)^2}+\frac{w_b^2+w_c^2}{(h_b+h_c)^2}+\frac{h_c^2+h_a^2}{(m_c+m_a)^2}\le\frac{3}{16}\cdot\left(\frac{1823}{64}\cdot\left(\frac{R}{r}\right)^6-1815\right)$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Soumava Chakraborty-Kolkata-India

$$\frac{m_a^2+m_b^2}{(w_a+w_b)^2}+\frac{w_b^2+w_c^2}{(h_b+h_c)^2}+\frac{h_c^2+h_a^2}{(m_c+m_a)}\le\sum_{cyc}\frac{m_b^2+m_c^2}{(h_b+h_c)^2}\overset{\substack{Panaitopol\\ and\\ A-G}}{\le}$$

$$\frac{R^2}{4r^2}\cdot\sum_{cyc}\frac{h_b^2+h_c^2}{4h_bh_c}=\frac{R^2}{16r^2}\sum_{cyc}\left(\frac{c}{b}+\frac{b}{c}\right)\overset{Bandila}{\le}\frac{3R^2}{16r^3}\ and\ \frac{3}{16}\cdot\left(\frac{1823}{64}\cdot\left(\frac{R}{r}\right)^6-1815\right)$$

$$\overset{Euler}{\ge}\frac{3}{16}\cdot\left(\frac{1823}{8}\cdot\left(\frac{R}{r}\right)^3-1815\right)\therefore\text{ it suffices to prove :}$$

$$\frac{3}{16}\cdot\left(\frac{1823R^3-8.1815r^3}{8r^3}\right)\ge\frac{3R^2}{16r^3}\Leftrightarrow 1815\left(R^3-8r^3\right)\ge 0\to\text{true via Euler}$$

$$\therefore\frac{m_a^2+m_b^2}{(w_a+w_b)^2}+\frac{w_b^2+w_c^2}{(h_b+h_c)^2}+\frac{h_c^2+h_a^2}{(m_c+m_a)^2}\le\frac{3}{16}\cdot\left(\frac{1823}{64}\cdot\left(\frac{R}{r}\right)^6-1815\right)$$

$\forall\,\Delta$ ABC,'' ='' iff Δ ABC is equilateral (QED)