ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC and $\forall$ n, m $\geq$ 0, the following relationship holds :

$$\frac{r_a^n}{(m_a+w_b)^m}+\frac{r_b^n}{(w_b+h_c)^m}+\frac{r_c^n}{(h_c+m_a)^m}\geq\frac{3^{n-m+1}.r^n}{R^m}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Soumava Chakraborty-Kolkata-India

$$\frac{r_a^n}{(m_a+w_b)^m}+\frac{r_b^n}{(w_b+h_c)^m}+\frac{r_c^n}{(h_c+m_a)^m}\geq$$

$$\frac{r_a^n}{(m_a+m_b)^m}+\frac{r_b^n}{(m_b+m_c)^m}+\frac{r_c^n}{(m_c+m_a)^m}\overset{A-G}{\geq}3.\sqrt[3]{\frac{\left(\prod_{cyc}r_a\right)^n}{\left(\prod_{cyc}(m_a+m_b)\right)^m}}$$

$$=3.\frac{\left(\sqrt[3]{rs^2}\right)^n}{\left(\sqrt[3]{(m_a+m_b)}\right)^m}\overset{\substack{\text{Mitrinovic}\\ \text{and}\\ A-G}}{\geq}3.\frac{\left(\sqrt[3]{r.27r^2}\right)^n}{\left(\frac{\sum_{cyc}(m_a+m_b)}{3}\right)^m}=3.\frac{(3r)^n}{\left(\frac{2}{3}\left(\sum_{cyc}m_a\right)\right)^m}$$

$$\overset{\text{Leuenberger + Euler}}{\geq}3.\frac{(3r)^n}{\left(\frac{2}{3}\left(\frac{9R}{2}\right)\right)^m}=\frac{3^{n-m+1}.r^n}{R^m}\ \forall\ \Delta\ ABC,$$

'' ='' iff Δ ABC is equilateral (QED)