ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\left(\frac{m_a}{w_b}+\frac{w_b}{h_c}\right)^2+\left(\frac{m_b}{w_c}+\frac{w_c}{h_a}\right)^2+\left(\frac{m_c}{w_a}+\frac{w_a}{h_b}\right)^2\geq\frac{192r^4}{3R^4-32r^4}$$

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Solution by Daniel Sitaru-Romania

$$\left(\frac{m_a}{w_b}+\frac{w_b}{h_c}\right)^2+\left(\frac{m_b}{w_c}+\frac{w_c}{h_a}\right)^2+\left(\frac{m_c}{w_a}+\frac{w_a}{h_b}\right)^2\overset{AM-GM}{\geq}$$

$$\geq\left(2\sqrt{\frac{m_a}{w_b}\cdot\frac{w_b}{h_c}}\right)^2+\left(2\sqrt{\frac{m_b}{w_c}\cdot\frac{w_c}{h_a}}\right)^2+\left(2\sqrt{\frac{m_c}{w_a}\cdot\frac{w_a}{h_b}}\right)^2=$$

$$=4\sum_{cyc}\frac{m_a}{h_c}=4\sum_{cyc}\frac{m_a}{\frac{2F}{c}}=\frac{2}{F}\sum_{cyc}cm_a\overset{CEBYSHEV}{\geq}$$

$$\geq\frac{2}{rs}\cdot\frac{1}{3}\sum_{cyc}a\sum_{cyc}m_a\overset{GOTMAN}{\geq}\frac{2}{rs}\cdot\frac{2s}{3}\cdot 9r=12$$

Remains to prove:

$$12\geq\frac{192r^4}{3R^4-32r^4}$$

$$12(3R^4-32r^4)\geq 192r^4$$

$$3R^4-32r^4\geq 16r^4\Leftrightarrow 3R^4\geq 48r^4\Leftrightarrow R^4\geq 16r^4\Leftrightarrow R\geq 2r\ (Euler)$$

Equality holds for: $a=b=c$.