# ROMANIAN MATHEMATICAL MAGAZINE

**In all acute triangles $ABC$ the following relationship holds:**

$$a(cotB+cotC)+b(cotC+cotA)+c(cotA+cotB)\ge 12r$$

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$$a(cotB+cotC)+b(cotC+cotA)+c(cotA+cotB)=\sum_{cyc}a(cotB+cotC)=$$

$$=\sum_{cyc}a\left(\frac{cosB}{sinB}+\frac{cosC}{sinC}\right)=\sum_{cyc}a\cdot\frac{cosBsinC+cosCsinB}{sinBsinC}=$$

$$=\sum_{cyc}a\cdot\frac{sin(B+C)}{sinBsinC}=\sum_{cyc}a\cdot\frac{sin(\pi-A)}{sinBsinC}=\sum_{cyc}a\cdot\frac{sinA}{sinBsinC}\overset{AM-GM}{\ge}$$

$$\ge 3\cdot\sqrt[3]{abc\cdot\frac{sinAsinBsinC}{sinBsinC\cdot sinCsinA\cdot sinAsinB}}=$$

$$=3\sqrt[3]{\frac{abc}{sinAsinBsinC}}=3\sqrt[3]{\frac{abc}{\frac{a}{2R}\cdot\frac{b}{2R}\cdot\frac{c}{2R}}}=$$

$$=3\sqrt[3]{8R^3}=6R\overset{EULER}{\ge}6\cdot 2r=12r$$

Equality holds for $a=b=c$.