ROMANIAN MATHEMATICAL MAGAZINE

In $\Delta ABC, n \in \mathbb{N}$ the following relationship holds:

$$min\left(\sum_{cyc}(b^n+c^n)tan\frac{A}{2}, \sum_{cyc}a^n\left(tan\frac{B}{2}+tan\frac{C}{2}\right)\right) \geq 2^{n+1}\cdot 3^{\frac{n+1}{2}}\cdot r^n$$

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$$\sum_{cyc}(b^n+c^n)tan\frac{A}{2} \overset{AM-GM}{\geq} 2\sum_{cyc}\sqrt{b^nc^n}\cdot tan\frac{A}{2} \overset{AM-GM}{\geq}$$

$$\geq 2\cdot 3\cdot\sqrt[3]{\sqrt{b^nc^n}\cdot\sqrt{c^na^n}\cdot\sqrt{a^nb^n}\cdot tan\frac{A}{2}tan\frac{B}{2}tan\frac{C}{2}} =$$

$$= 6\cdot\sqrt[3]{(abc)^n\cdot\frac{r}{s}} = 6\cdot\sqrt[3]{4^n\cdot R^n\cdot r^n\cdot s^n\cdot\frac{r}{s}} =$$

$$= 6\cdot\sqrt[3]{4^n\cdot R^n\cdot r^{n+1}\cdot s^{n-1}} \overset{EULER}{\geq} 6\cdot\sqrt[3]{2^{2n}\cdot 2^n\cdot r^n\cdot r^{n+1}\cdot s^{n-1}} =$$

$$= 6\cdot 2^n\cdot\sqrt[3]{r^{2n+1}\cdot s^{n-1}} \overset{MITRINOVIC}{\geq} 3\cdot 2^{n+1}\cdot\sqrt[3]{r^{2n+1}\cdot(3\sqrt{3}r)^{n-1}} =$$

$$= 2^{n+1}\cdot 3\cdot 3^{\frac{n}{2}}\cdot\sqrt[3]{r^{2n+1+n-1}} = 2^{n+1}\cdot 3^{\frac{n+1}{2}}\cdot r^n$$

$$\sum_{cyc}a^n\left(tan\frac{B}{2}+tan\frac{C}{2}\right) \overset{AM-GM}{\geq} 2\sum_{cyc}a^n\sqrt{tan\frac{B}{2}tan\frac{C}{2}} \overset{AM-GM}{\geq}$$

$$\geq 2\cdot 3\cdot\sqrt[3]{(abc)^n\cdot\prod_{cyc}\sqrt{tan\frac{B}{2}tan\frac{C}{2}}} = 6\cdot\sqrt[3]{(4Rrs)^n\cdot\frac{r}{s}} = 6\cdot\sqrt[3]{4^n\cdot R^n\cdot r^n\cdot s^n\cdot\frac{r}{s}} =$$

$$= 6\cdot\sqrt[3]{4^n\cdot R^n\cdot r^{n+1}\cdot s^{n-1}} \overset{EULER}{\geq} 6\cdot\sqrt[3]{2^{2n}\cdot 2^n\cdot r^n\cdot r^{n+1}\cdot s^{n-1}} =$$

$$= 6\cdot 2^n\cdot\sqrt[3]{r^{2n+1}\cdot s^{n-1}} \overset{MITRINOVIC}{\geq} 3\cdot 2^{n+1}\cdot\sqrt[3]{r^{2n+1}\cdot(3\sqrt{3}r)^{n-1}} =$$

$$= 2^{n+1}\cdot 3\cdot 3^{\frac{n}{2}}\cdot\sqrt[3]{r^{2n+1+n-1}} = 2^{n+1}\cdot 3^{\frac{n+1}{2}}\cdot r^n$$

Equality holds for $a = b = c$.