ROMANIAN MATHEMATICAL MAGAZINE

In any Δ ABC and $\forall$ m, n $\in \mathbb{N}$; n $\geq$ 2, the following relationship ho$l$ds :

$$\frac{(h_a^m + w_b^m + m_c^m)^n}{r_a} + \frac{(h_b^m + w_c^m + m_a^m)^n}{r_b} + \frac{(h_c^m + w_a^m + m_b^m)^n}{r_c} \geq \frac{2.3^{n(m+1)}.r^{mn}}{R}$$

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Solution by Soumava Chakraborty-Kolkata-India

$$\frac{(h_a^m + w_b^m + m_c^m)^n}{r_a} + \frac{(h_b^m + w_c^m + m_a^m)^n}{r_b} + \frac{(h_c^m + w_a^m + m_b^m)^n}{r_c} \overset{\text{Holder}}{\geq}$$

$$\frac{\left(\sum_{cyc} h_a^m + \sum_{cyc} w_a^m + \sum_{cyc} m_a^m\right)^n}{3^{n-2}\left(\sum_{cyc} r_a\right)} \geq \frac{\left(\sum_{cyc} h_a^m + \sum_{cyc} h_a^m + \sum_{cyc} h_a^m\right)^n}{3^{n-2}(4R + r)} \overset{\substack{\text{Holder}\\ and\\ \text{Euler}}}{\geq}$$

$$\frac{3^n\left(\frac{1}{3^{m-1}}\left(\sum_{cyc} h_a\right)^m\right)^n}{3^{n-2}.\frac{9R}{2}} = \frac{2}{3^{mn-n}.R}\left(\sum_{cyc}\frac{2rs}{a}\right)^{mn} \overset{\text{Bergstrom}}{\geq} \frac{2}{3^{mn-n}.R}\left(\frac{2rs.9}{2s}\right)^{mn}$$

$$= \frac{2r^{mn}.3^{2mn}}{3^{mn-n}.R} = \frac{2.3^{n(m+1)}.r^{mn}}{R} \;\; \forall \, \Delta \, ABC, '' ='' \text{ iff } \Delta \, ABC \text{ is equil}ateral \text{ (QED)}$$