ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$(b+c)cot\frac{A}{2}+(c+a)cot\frac{B}{2}+(a+b)cot\frac{C}{2}\ge 36r$$

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Solution by Daniel Sitaru-Romania

$$(b+c)cot\frac{A}{2}+(c+a)cot\frac{B}{2}+(a+b)cot\frac{C}{2}=\sum_{cyc}(a+b)cot\frac{C}{2}\overset{AM-GM}{\ge}$$

$$\ge 3\cdot\sqrt[3]{\prod_{cyc}(a+b)\cdot\prod_{cyc}cot\frac{A}{2}}\overset{CESARO}{\ge}3\cdot\sqrt[3]{8abc\cdot\frac{s}{r}}=$$

$$=3\cdot\sqrt[3]{32Rrs\cdot\frac{s}{r}}\overset{EULER}{\ge}3\cdot\sqrt[3]{64rs^2}=12\sqrt[3]{rs^2}\overset{MITRINOVIC}{\ge}12\sqrt[3]{27r^3}=36r$$

Equality holds for $a=b=c$.