ROMANIAN MATHEMATICAL MAGAZINE

In acute ΔABC the following relationship holds:

$$(b+c)secA+(c+a)secB+(a+b)secC \geq 24\sqrt{3}r$$

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$$(b+c)secA+(c+a)secB+(a+b)secC \overset{AM-GM}{\geq}$$

$$\geq 3\cdot\sqrt[3]{\prod_{cyc}(a+b)\cdot\prod_{cyc}\frac{1}{cosA}} \overset{CESARO}{\geq} 3\cdot\sqrt[3]{8abc\cdot\prod_{cyc}\frac{1}{cosA}} \geq$$

$$\geq 6\cdot\sqrt[3]{abc\cdot\frac{1}{\frac{1}{8}}} = 12\cdot\sqrt[3]{abc} = 12\cdot\sqrt[3]{4Rrs} \overset{EULER}{\geq} 12\cdot\sqrt[3]{8r^2s} \geq$$

$$\overset{MITRINOVIC}{\geq} 24\cdot\sqrt[3]{r^2\cdot 3\sqrt{3}r} = 24r\cdot\sqrt[3]{\left(\sqrt{3}\right)^3} = 24\sqrt{3}r$$

Equality holds for $a=b=c$.