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In $\Delta\, ABC$ the following relationship holds:

$$a^2\cdot\left(cot\frac{B}{2}+cot\frac{C}{2}\right)+b^2\cdot\left(cot\frac{C}{2}+cot\frac{A}{2}\right)+c^2\cdot\left(cot\frac{A}{2}+cot\frac{B}{2}\right)\geq 72\sqrt{3}r^2$$

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Solution by Mirsadix Muzefferov-Azerbaijan

$$a^2\cdot\left(cot\frac{B}{2}+cot\frac{C}{2}\right)+b^2\cdot\left(cot\frac{C}{2}+cot\frac{A}{2}\right)+c^2\cdot\left(cot\frac{A}{2}+cot\frac{B}{2}\right)=$$

$$=a^2\cdot\frac{sin\frac{B+C}{2}}{sin\frac{B}{2}\cdot sin\frac{C}{2}}+b^2\cdot\frac{sin\frac{A+C}{2}}{sin\frac{A}{2}\cdot sin\frac{C}{2}}+c^2\cdot\frac{sin\frac{A+B}{2}}{sin\frac{A}{2}\cdot sin\frac{B}{2}}=$$

$$=a^2\cdot\frac{cos\frac{A}{2}}{sin\frac{B}{2}\cdot sin\frac{C}{2}}+b^2\cdot\frac{cos\frac{B}{2}}{sin\frac{A}{2}\cdot sin\frac{C}{2}}+c^2\cdot\frac{cos\frac{C}{2}}{sin\frac{A}{2}\cdot sin\frac{B}{2}}=$$

$$=\frac{a^2sin\frac{A}{2}\cdot cos\frac{A}{2}+b^2sin\frac{B}{2}+cos\frac{B}{2}+c^2sin\frac{C}{2}\cdot cos\frac{C}{2}}{sin\frac{A}{2}\cdot sin\frac{B}{2}\cdot sin\frac{C}{2}}\geq$$

$$sin\frac{A}{2}\cdot sin\frac{B}{2}\cdot sin\frac{C}{2}\leq\frac{1}{8}\qquad true\ \ (1)$$

$$a=2RsinA\,;\ b=2RsinB\,;\ c=2RsinC\ \ (2)$$

$$S=2R^2\cdot sinA\cdot sinB\cdot sinC\qquad(3)$$

$$S\geq 3\sqrt{3}r^2\ \ (4)-Mitrinovic$$

$$\overset{(1)}{\geq}4(a^2sinA+b^2sinB+c^2sinC)\overset{A-G}{\geq}4\cdot 3\cdot\sqrt[3]{(abc)^2sinA\cdot sinB\cdot sinC}\overset{(2)}{=}$$

$$=12\sqrt[3]{64R^6(sinA\cdot sinB\cdot sinC)^3}=48R^2\cdot sinA\cdot sinB\cdot sinC\overset{(3)}{=}48R^2\cdot\frac{S}{2R^2}=$$

$$=24S\overset{(4)}{\geq}72\sqrt{3}r^2$$

Equality holds for a=b=c.