ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\frac{a}{b(\sin A+\sin B)+c\sin C}+\frac{b}{c(\sin A+\sin B)+a\sin C}+\frac{c}{a(\sin A+\sin B)+a\sin C}\geq\frac{2}{\sqrt{3}}$$

Proposed by Zaza Mzhavanadze-Georgia

Solution by Tapas Das-India

$$\sum\sin A=\frac{a+b+c}{2R}=\frac{2s}{2R}=\frac{s}{R}\overset{Mitrinovic}{\leq}3\sqrt{3}\frac{R}{2R}=\frac{3\sqrt{3}}{2}\quad(1)$$

$$\sum\frac{a}{b(\sin A+\sin B)+c\sin C}=\sum\frac{a^2}{ba(\sin A+\sin B)+ca\sin C}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(a+b+c)^2}{(\sum ab)(\sum\sin A)}\overset{3\sum ab\leq(\sum a)^2}{\geq}\frac{(a+b+c)^2}{\frac{(\sum a)^2}{3}(\sum\sin A)}\overset{(1)}{\geq}\frac{3}{\frac{3\sqrt{3}}{2}}=\frac{2}{\sqrt{3}}$$

Equality holds for $a=b=c$