ROMANIAN MATHEMATICAL MAGAZINE

In ΔABC the following relationship holds:

$$\sum_{cyc}\frac{a}{b(\sin^2 A+\sin^2 B)+c\sin^2 C}\geq\frac{4}{3}$$

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Solution by Tapas Das-India

$$\sum\sin^2 A=\left(\sum\sin A\right)^2-2\sum\sin A\sin B=\left(\frac{s}{R}\right)^2-\frac{2(s^2+r^2+4Rr)}{4R^2}=$$

$$=\frac{2s^2-2r^2-8Rr}{4R^2}\overset{Gerretsen}{\leq}\frac{8R^2+4r^2}{4R^2}=2+\left(\frac{r}{R}\right)^2\overset{Euler}{\leq}\frac{9}{4}\ (1)$$

$$\sum\frac{a}{b(\sin^2 A+\sin^2 B)+c\sin^2 C}=\sum\frac{a^2.}{ba(\sin^2 A+\sin^2 B)+ca\sin^2 C}\overset{Bergstrom}{\geq}$$

$$\geq\frac{(\sum a)^2}{(\sum ab)(\sum\sin^2 A)}\overset{3\sum ab\leq(\sum a)^2\&(1)}{\geq}\frac{(\sum a)^2}{\frac{(\sum a)^2}{3}\frac{9}{4}}=\frac{4}{3}$$

$$Equality\ for\ a=b=c.$$