Find:

$$\Omega = \int_0^1 \int_0^1 \int_0^1 \frac{dxdydz}{\sqrt{x+2y+3z}}$$

Proposed by Ankush Kumar Parcha-India

Solution by Mirsadix Muzefferov-Azerbaijan

$$\int_0^1 \int_0^1 \int_0^1 \frac{dxdydz}{\sqrt{x+2y+3z}} = \int_0^1 \int_0^1 (2\sqrt{x+2y+3z})\,\Big|_0^1\, dydz = \int_0^1 (2\sqrt{x+2y+3z}) - \sqrt{2y+3z})dy\,\Big|_0^1\, dz$$

$$\int_0^1 2\left(\frac{(2y+3z+1)^{\frac{3}{2}}}{\frac{3}{2}} \cdot \frac{1}{2} - \frac{(2y+3z)^{\frac{3}{2}}}{\frac{3}{2}} \cdot \frac{1}{2} \right)\Big|_0^1\, dz$$

$$= \int_0^1 \left[\frac{2}{3}(3+3z)^{\frac{3}{2}} - \frac{2}{3}(3z+2)^{\frac{3}{2}} - (\frac{2}{3}(3z+1)^{\frac{3}{2}} - \frac{2(3z)^{\frac{3}{2}}}{3} \right] dz$$

$$\frac{2}{3}\int_0^1 \left[(3+3z)^{\frac{3}{2}} - (3z+2)^{\frac{3}{2}} - ((3z+1)^{\frac{3}{2}} - (3z)^{\frac{3}{2}}) \right] dz = \frac{2}{3}\left[\frac{(3+3z)^{\frac{5}{2}}}{\frac{5}{2}} \cdot \frac{1}{3} - \frac{(3z+2)^{\frac{5}{2}}}{\frac{5}{2}} \cdot \frac{1}{3} \right]\Big|_0^1 =$$

$$\frac{4}{45}\left(36\sqrt{6} - 25\sqrt{5} - 16\sqrt{4} + 9\sqrt{3} - 9\sqrt{3} + 4\sqrt{2} + 1\right) = \frac{4}{45}\left(36\sqrt{6} - 25\sqrt{5} + 4\sqrt{2} - 31\right)$$