ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\sin(x)sin^2(2y)sin^3(3z)}{xy^2z^3}dxdydz$$

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Solution by Shirvan Tahirov-Azerbaijan

$$\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\int_{-\infty}^{\infty}\frac{\sin(x)sin^2(2y)sin^3(3z)}{xy^2z^3}dxdydz$$

$$=\int_{-\infty}^{\infty}\frac{\sin(x)}{x}dx.\int_{-\infty}^{\infty}\frac{sin^2(2y)}{y^2}dy.\int_{-\infty}^{\infty}\frac{sin^3(3z)}{z^3}dz$$

$$\{*\int_{-\infty}^{\infty}\frac{\sin(x)}{x}dx.\int_{-\infty}^{\infty}\frac{sin^2(2y)}{y^2}dy.\int_{-\infty}^{\infty}\frac{sin^3(3z)}{z^3}dz=M\ .\ N\ .\ K*\}$$

$$M=\int_{-\infty}^{\infty}\frac{\sin(x)}{x}dx=2.\int_{0}^{\infty}\frac{\sin(x)}{x}dx=2.\frac{\pi}{2}=\pi$$

$$N=\int_{-\infty}^{\infty}\frac{sin^2(2y)}{y^2}dy=2\int_{0}^{\infty}\frac{sin^2(2y)}{y^2}dy=4.\frac{\pi}{2}=2\pi$$

$$K=\int_{-\infty}^{\infty}\frac{sin^3(3z)}{z^3}dz$$

$$=2\int_{0}^{\infty}\frac{sin^3(3z)}{z^3}dz\overset{3z=t,\frac{dt}{dz}=3}{=}18\int_{0}^{\infty}\frac{sin^3(t)}{t^3}dt=\frac{81}{4}\int_{0}^{\infty}\frac{\sin(3t)}{t}dt-$$

$$-\frac{27}{4}\int_{0}^{\infty}\frac{\sin(t)}{t}dt=\frac{27}{4}(3.\frac{\pi}{2}-\frac{\pi}{2})=\frac{27}{4}\pi$$

$$\int_{-\infty}^{\infty}\frac{\sin(x)}{x}dx.\int_{-\infty}^{\infty}\frac{sin^2(2y)}{y^2}dy.\int_{-\infty}^{\infty}\frac{sin^3(3z)}{z^3}dz=M\ .\ N\ .\ K=\pi.2\pi.\frac{27}{4}\pi=\frac{27}{2}\pi^3$$