ROMANIAN MATHEMATICAL MAGAZINE

Find a closed form:

$$\int_0^1\int_0^1 \frac{\arcsin(x)+\arccos(y)}{\sqrt{xy}}\,dxdy$$

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$$\int_0^1\int_0^1 \frac{\arcsin(x)+\arccos(y)}{\sqrt{xy}}\,dxdy=\int_0^1\int_0^1 \frac{\arcsin(x)}{\sqrt{xy}}\,dxdy+\int_0^1\int_0^1 \frac{\arccos(y)}{\sqrt{xy}}\,dxdy=M+K$$

$$M=\int_0^1\int_0^1 \frac{\arcsin(x)}{\sqrt{xy}}\,dxdy=\int_0^1 \frac{1}{\sqrt{y}}dy\int_0^1 \frac{\arcsin(x)}{\sqrt{x}}dx=2\int_0^1 \frac{\arcsin(x)}{\sqrt{x}}dx \overset{I.B.P}{=}$$

$$2\Big|_0^1 2\sqrt{x}\arcsin(x)-4\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x^2}}dx$$

$$K=\int_0^1\int_0^1 \frac{\arccos(y)}{\sqrt{xy}}\,dxdy=\int_0^1 \frac{\arccos(y)}{\sqrt{y}}dy\int_0^1 \frac{1}{\sqrt{x}}dx=2\int_0^1 \frac{\arccos(y)}{\sqrt{y}}dx \overset{I.B.P}{=}$$

$$2\Big|_0^1 2\sqrt{x}\arccos(y)+4\int_0^1 \frac{\sqrt{y}}{\sqrt{1-y^2}}dy=4\int_0^1 \frac{\sqrt{y}}{\sqrt{1-y^2}}dy$$

$$M+K=2\pi-4\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x^2}}dx+4\int_0^1 \frac{\sqrt{y}}{\sqrt{1-y^2}}dy=2\pi$$