ROMANIAN MATHEMATICAL MAGAZINE

Prove that:

$$\int_0^\infty \frac{1}{(1+x^{2025})(1+x^2)}dx = \frac{\pi}{2}$$

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$$Let\ I = \int_0^\infty \frac{1}{(1+x^{2025})(1+x^2)}dx \quad (A)$$

$$Let\ x = \frac{1}{t}\ then\ dx = -\frac{1}{t^2}dt$$

$$I = \int_\infty^0 \frac{-(t^{2025}.t^2)}{(1+t^{2025})(1+t^2)t^2}dt\ = \int_0^\infty \frac{(t^{2025}.)}{(1+t^{2025})(1+t^2)}dt$$

$$= \int_0^\infty \frac{(x^{2025}.)}{(1+x^{2025})(1+x^2)}dx\ (B)$$

$$adding\ (A) and\ (B)\ we\ get:$$

$$2I = \int_0^\infty \frac{(x^{2025}+1.)}{(1+x^{2025})(1+x^2)}dx = \int_0^\infty \frac{(x^{2025}+1)}{(1+x^{2025})(1+x^2)}dx = \int_0^\infty \frac{1}{(1+x^2)}dx =$$

$$= \lim_{x\to\infty}\int_0^x \frac{1}{(1+x^2)}dx = \lim_{x\to\infty}[\tan^{-1}x]_0^x = \tan^{-1}\infty = \frac{\pi}{2}$$

$$I = \frac{\pi}{4}$$