

ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$I = \int \left(\frac{\cos\left(\frac{\pi}{3} - x\right) \sin(3x) \cos\left(\frac{\pi}{3} + x\right)}{\sin\left(\frac{\pi}{3} - x\right) \cos(3x) \sin\left(\frac{\pi}{3} + x\right)} \right)^2 dx$$

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Solution by Togrul Ehmedov-Azerbaijan

$$\begin{aligned} I &= \int \left(\frac{\cos\left(\frac{\pi}{3} - x\right) \sin(3x) \cos\left(\frac{\pi}{3} + x\right)}{\sin\left(\frac{\pi}{3} - x\right) \cos(3x) \sin\left(\frac{\pi}{3} + x\right)} \right)^2 dx \\ &= \int \left(\frac{\cos\left(\frac{\pi}{3} - x\right) \cos(x) \cos\left(\frac{\pi}{3} + x\right) \sin(3x)}{\sin\left(\frac{\pi}{3} - x\right) \sin(x) \sin\left(\frac{\pi}{3} + x\right) \cos(3x)} \right)^2 \left(\frac{\sin(x)}{\cos(x)} \right)^2 dx \\ &= \int \left(\frac{\tan(3x)}{\tan\left(\frac{\pi}{3} - x\right) \tan(x) \tan\left(\frac{\pi}{3} + x\right)} \right)^2 (\tan(x))^2 dx \end{aligned}$$

We know that $\tan\left(\frac{\pi}{3} - x\right) \tan(x) \tan\left(\frac{\pi}{3} + x\right) = \tan(3x)$

Then we can write

$$I = \int \left(\frac{\tan(3x)}{\tan(3x)} \right)^2 (\tan(x))^2 dx = \int (\tan(x))^2 dx = \tan(x) - x + C$$