ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \int x^{\log_2 x} \cdot (1 + 2\log_2 x)\, dx$$

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$$u = x^{\log_2 x + 1}, \qquad \log_2 u = \log_2 x^{\log_2 x + 1}$$

$$\log_2 u = (\log_2 x + 1)\log_2 x, \qquad \log_2 u = (\log_2 x)^2 + \log_2 x$$

$$\frac{du}{u\ln 2} = \left(2\log_2 x \cdot \frac{1}{x\ln 2} + \frac{1}{x\ln 2}\right) dx, \qquad \frac{du}{u} = \left(2\log_2 x \cdot \frac{1}{x} + \frac{1}{x}\right) dx$$

$$du = \frac{u(1 + 2\log_2 x)}{x} dx, \qquad du = \frac{x^{\log_2 x + 1}(1 + 2\log_2 x)}{x} dx$$

$$du = x^{\log_2 x} \cdot (1 + 2\log_2 x) dx$$

$$\Omega = \int x^{\log_2 x} \cdot (1 + 2\log_2 x)\, dx = \int du = u + C = x^{\log_2 x + 1} + C$$