ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_0^{12\pi}\frac{x\left(3\cos\frac{x}{2}-2\sin\frac{x}{3}\right)}{1+\left(\sin\frac{x}{2}+\cos\frac{x}{3}\right)^2}dx$$

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Solution by Ravi Prakash-New Delhi-India

Using $\int_0^a f(x)\,dx=\int_0^a f(a-x)\,dx$,

we get

$$\Omega=\int_0^{12\pi}\frac{(12\pi-x)\left[3\cos\left(6\pi-\frac{x}{2}\right)+2\cos\left(4\pi-\frac{x}{3}\right)\right]dx}{1+\left[\sin\left(6\pi-\frac{x}{2}\right)+\sin\left(4\pi-\frac{x}{3}\right)\right]^2}$$

$$=\int_0^{12\pi}\frac{(12\pi-x)\left[3\cos\frac{x}{2}+2\cos\frac{x}{3}\right]}{1+\left[-\sin\frac{x}{2}-\sin\frac{x}{3}\right]^2}dx=12\pi\Omega_1=\Omega$$

$2\Omega=12\pi\Omega_1$, or $\Omega=6\pi\Omega_1$

where

$$\Omega_1=\int_0^{12\pi}\frac{3\cos\left(\frac{x}{2}\right)+2\cos\left(\frac{x}{3}\right)}{1+\left[\sin\frac{x}{2}+\sin\frac{x}{3}\right]^2}dx=6\int_0^{12\pi}\frac{\frac{1}{2}\cos\frac{x}{2}+\frac{1}{3}\cos\frac{x}{3}}{1+\left[\sin\frac{x}{2}+\sin\frac{x}{3}\right]^2}dx$$

$$=6\left(\tan^{-1}\left[\sin\frac{x}{2}+\sin\frac{x}{3}\right]\right)\Big]_0^{12\pi}=6[0]=0$$

Thus, $\Omega=0$