ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega(x)=\int_1^e \frac{\sqrt{\log x}}{\sqrt{\log(1+e-x)}+\sqrt{\log x}}dx$$

Proposed by Khaled Abd Imouti-Syria

Solution by Daniel Sitaru-Romania

$$\Omega(x)=\int_1^e \frac{\sqrt{\log x}}{\sqrt{\log(1+e-x)}+\sqrt{\log x}}dx$$

$$y=1+e-x,\qquad dx=-dy$$

$$x=1\Rightarrow y=e$$

$$x=e\Rightarrow y=1$$

$$\Omega(x)=-\int_e^1 \frac{\sqrt{\log(1+e-y)}}{\sqrt{\log\big(1+e-(1+e-y)\big)}+\sqrt{\log(1+e-y)}}(-dy)=$$

$$=\int_1^e \frac{\sqrt{\log(1+e-y)}}{\sqrt{\log y}+\sqrt{\log(1+e-y)}}dy=\int_1^e \frac{\sqrt{\log(1+e-x)}}{\sqrt{\log x}+\sqrt{\log(1+e-x)}}dx$$

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$$2\Omega(x)=\int_1^e \frac{\sqrt{\log x}}{\sqrt{\log(1+e-x)}+\sqrt{\log x}}dx+\int_1^e \frac{\sqrt{\log(1+e-x)}}{\sqrt{\log x}+\sqrt{\log(1+e-x)}}dx$$

$$2\Omega(x)=\int_1^e \frac{\sqrt{\log x}+\sqrt{\log(1+e-x)}}{\sqrt{\log(1+e-x)}+\sqrt{\log x}}dx$$

$$2\Omega(x)=\int_1^e dx$$

$$2\Omega(x)=e-1$$

$$\Omega(x)=\frac{e-1}{2}$$