ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos^3(x)+\cos^5(x)}{\sin^2(x)+\sin^4(x)}dx$$

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$$\frac{\cos^3(x)+\cos^5(x)}{\sin^2(x)+\sin^4(x)}=\cos(x).\frac{\cos^2(x)+\cos^4(x)}{\sin^2(x)+\sin^4(x)}=\cos(x).\frac{1-\sin^2(x)+(1-\sin^2(x))^2}{\sin^2(x)+\sin^4(x)}=$$

$$\cos(x).\frac{\sin^4(x)-3\sin^2(x)+2}{\sin^2(x)+\sin^4(x)}=\cos(x).\left(1-\frac{4\sin^2(x)-2}{\sin^2(x)+\sin^4(x)}\right)=$$

$$\cos(x).\left(1-\frac{4\sin^2(x)-2}{\sin^2(x)\left(1+\sin^2(x)\right)}\right)=\cos(x).(1-A)$$

$$A=\frac{4\sin^2(x)-2}{\sin^2(x)\left(1+\sin^2(x)\right)}=-\frac{2}{\sin^2(x)}+\frac{6}{1+\sin^2(x)}\cos(x).(1-A)$$

$$\to\cos(x).\left(1+\frac{2}{\sin^2(x)}-\frac{6}{1+\sin^2(x)}\right)=\cos(x)+\frac{2\cos(x)}{\sin^2(x)}-\frac{6\cos(x)}{1+\sin^2(x)}=$$

$$\Omega=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos^3(x)+\cos^5(x)}{\sin^2(x)+\sin^4(x)}dx=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\left(\cos(x)+\frac{2\cos(x)}{\sin^2(x)}-\frac{6\cos(x)}{1+\sin^2(x)}\right)dx=$$

$$\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\cos(x)\,dx+2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos(x)}{\sin^2(x)}dx$$

$$-6\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos(x)}{1+\sin^2(x)}dx=\sin(x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}+2\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{d(\sin(x))}{\sin^2(x)}-6\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{d(\sin(x))}{1+\sin^2(x)}=$$

$$\sin(x)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}-2\left(\frac{1}{\sin(x)}\right)\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}-6(\arctan(\sin(x))\Big|_{\frac{\pi}{6}}^{\frac{\pi}{2}}=$$

$$=\left(1-\frac{1}{2}\right)-2(1-2)-6(\frac{\pi}{4}-\arctan\left(\frac{1}{2}\right)=\frac{5-3\pi}{2}+6\arctan\left(\frac{1}{2}\right)$$

$$\Omega=\int_{\frac{\pi}{6}}^{\frac{\pi}{2}}\frac{\cos^3(x)+\cos^5(x)}{\sin^2(x)+\sin^4(x)}dx=\frac{5-3\pi}{2}+6\arctan\left(\frac{1}{2}\right)$$