Find:

$$\int_{\frac{3\pi}{4}}^{\pi} \left(\frac{e^{\frac{1}{x}}}{x^2} + x \left(\frac{x}{cos^2(x)} + 2\tan(x) \right) \right) dx$$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Kartick Chandra Betal-India

$$\int_{\frac{3\pi}{4}}^{\pi} \left(\frac{e^{\frac{1}{x}}}{x^2} + x \left(\frac{x}{cos^2(x)} + 2\tan(x) \right) \right) dx =$$

$$= \left[-e^{\frac{1}{x}} \right]_{\frac{3\pi}{4}}^{\pi} + \int_{\frac{3\pi}{4}}^{\pi} x^2 sec^2(x)dx + 2\int_{\frac{3\pi}{4}}^{\pi} xtan(x)dx = \left(e^{\frac{4}{3\pi}} - e^{\frac{1}{\pi}} \right) + \left[x^2\tan(x) \right]_{\frac{3\pi}{4}}^{\pi} -$$

$$-2\int_{\frac{3\pi}{4}}^{\pi} xtan(x)dx + 2\int_{\frac{3\pi}{4}}^{\pi} xtan(x)dx = \left(e^{\frac{4}{3\pi}} - e^{\frac{1}{\pi}} \right) - \left(\frac{3\pi}{4} \right)^2 \tan\left(\pi - \frac{\pi}{4} \right) =$$

$$= \left(e^{\frac{4}{3\pi}} - e^{\frac{1}{\pi}} \right) + \frac{9}{16}\pi^2$$