

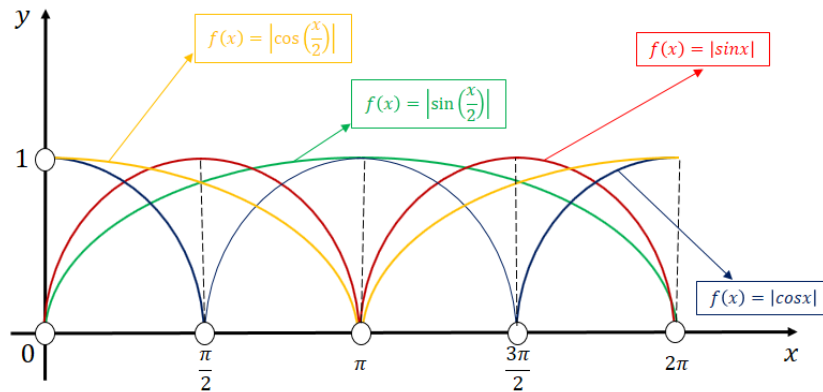
ROMANIAN MATHEMATICAL MAGAZINE

Prove that

$$I = \int_0^{2\pi} \left(|\sin x| + |\cos x| + \left| \sin \left(\frac{x}{2} \right) \right| + \left| \cos \left(\frac{x}{2} \right) \right| \right) dx = 16$$

Proposed by Nguyen Van Canh-Vietnam

Solution by Togrul Ehmedov-Azerbaijan



$$I_1 = \int_0^{2\pi} |\sin x| dx = 2 \int_0^{\pi} \sin x dx = 4$$

$$I_2 = \int_0^{2\pi} |\cos x| dx = 4 \int_0^{\frac{\pi}{2}} \cos x dx = 4$$

$$I_3 = \int_0^{2\pi} \left| \sin \left(\frac{x}{2} \right) \right| dx = \int_0^{2\pi} \sin \left(\frac{x}{2} \right) dx = 4$$

$$I_4 = \int_0^{2\pi} \left| \cos \left(\frac{x}{2} \right) \right| dx = 2 \int_0^{\pi} \cos \left(\frac{x}{2} \right) dx = 4$$

$$I = I_1 + I_2 + I_3 + I_4 = 16$$