ROMANIAN MATHEMATICAL MAGAZINE

**Find:**

$$\int_{-1}^{1}\frac{xln(x)}{x^4+x^2+1}dx$$

*Proposed by Shirvan Tahirov-Azerbaijan*

## *Solution 1 by Amin Hajiyev-Azerbaijan*

$$\Omega=\int_{-1}^{1}\frac{xlog(x)}{x^4+x^2+1}dx=\int_0^1\frac{xlog(x)}{x^4+x^2+1}dx+\int_{-1}^{0}\frac{xlog(x)}{x^4+x^2+1}dx=\int_0^1\frac{xlog(x)}{x^4+x^2+1}dx+\int_0^1\frac{i\pi x+xln(x)}{x^4+x^2+1}dx=$$

$$-\pi i\int_0^1\frac{x}{x^4+x^2+1}dx=-i\pi\int_0^1\frac{x-x^3}{1-x^6}dx=-i\pi\sum_{n=0}^{\infty}\left(\int_0^1 x^{6n+1}dx-\int_0^1 x^{6n+3}dx\right)=$$

$$-\pi i\sum_{n=0}^{\infty}\left(\frac{1}{6n+2}-\frac{1}{6n+4}\right)=-\frac{i\pi}{2}\sum_{n=0}^{\infty}\frac{1}{(3n+1)(3n+2)}=-\frac{i\pi}{18}\sum_{n=0}^{\infty}\frac{1}{(n+\frac{1}{3})(n+\frac{2}{3})}=$$

$$-\frac{i\pi}{18}\frac{\psi_0\left(\frac{2}{3}\right)-\psi_0\left(\frac{1}{3}\right)}{\frac{1}{3}}=-\frac{i\pi}{6}\pi\cot\left(\frac{\pi}{3}\right)=-\frac{i\pi^2}{6\sqrt{3}}$$

We know - $\psi_0(1-x)-\psi_0(x)=\pi\cot(\pi x)$

## *Solution 2 by Bui Hong Suc-Vietnam*

$$\Omega=\int_{-1}^{1}\frac{xln(x)}{x^4+x^2+1}dx=\overbrace{\int_{-1}^{0}\frac{xln(x)}{x^4+x^2+1}dx}^{x\to -x}+\int_0^1\frac{xln(x)}{x^4+x^2+1}dx=\int_1^0\frac{xln(-x)}{x^4+x^2+1}dx+\int_0^1\frac{xln(x)}{x^4+x^2+1}dx=$$

$$-\int_0^1\frac{x\{ln(-1)+\ln(x)\}}{x^4+x^2+1}dx+\int_0^1\frac{xln(x)}{x^4+x^2+1}dx=-\int_0^1\frac{xln(e^{i\pi})}{x^4+x^2+1}dx-\int_0^1\frac{xln(x)}{x^4+x^2+1}dx+\int_0^1\frac{xln(x)}{x^4+x^2+1}dx=$$

$$-i\pi\int_0^1\frac{x}{(x^2+\frac{1}{2})^2+\frac{3}{4}}dx=-\frac{i\pi}{\sqrt{3}}\arctan\left(\frac{2x^2+1}{\sqrt{3}}\right)\Big|_0^1=-\frac{i\pi}{\sqrt{3}}\left(\frac{\pi}{3}-\frac{\pi}{6}\right)=-\frac{i\pi^2}{6\sqrt{3}}$$