ROMANIAN MATHEMATICAL MAGAZINE

$$Ci(x)=\gamma+\log x+\sum_{n=1}^{\infty}\frac{(-x^2)^n}{2n\cdot(2n)!}$$

Prove that:

$$\int_a^b\left(\frac{\cos x}{x}\right)^2dx+\int_a^b Ci^2(x)\,dx\ge Ci^2(b)-Ci^2(a),\qquad 0<a\le b$$

Proposed by Daniel Sitaru – Romania

Solution by Hikmat Mammadov – Azerbaijan

$$Ci(x)=\gamma+\log(x)+\sum_{n=1}^{\infty}\frac{(-x^2)^n}{2n\cdot(2n)!}$$

$$\int_a^b\left(\frac{\cos x}{x}\right)^2dx+\int_a^b Ci^2(x)\,dx\ge Ci^2(b)-Ci^2(a)\to 0<a\le b$$

$$\int_a^b\left(f(x)^2+f'(x)^2\right)dx\ge\int_a^b 2f(x)f'(x)dx=f(b)^2-f(a)^2$$

$$\Rightarrow f(x)=Ci(x)\Rightarrow f'(x)=\frac{\cos x}{x}$$

$$\Rightarrow\int_a^b\left(Ci(x)^2+\left(\frac{\cos x}{x}\right)^2\right)dx\ge f(b)^2-f(a)^2$$