ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \lim_{n\to\infty}\left(n\int_0^1 x^n \exp(x)\,dx\right)$$

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Solution by Dimitris Kastriotis – Greece

$$n\cdot\int_0^1 x^n e^x\,dx = n\cdot\int_0^1 x^n\sum_{k=0}^{\infty}\frac{x^k}{k!}dx = n\cdot\int_0^1\sum_{k=0}^{\infty}\frac{x^{k+n}}{k!}dx =$$

$$= n\cdot\sum_{k=0}^{\infty}\frac{1}{k!}\int_0^1 x^{n+k}\,dx = n\cdot\sum_{k=0}^{\infty}\left(\frac{1}{k!}\cdot\frac{1}{n+k+1}\right) =$$

$$= \sum_{k=0}^{\infty}\left(\frac{1}{k!}\cdot\frac{1}{1+\frac{k+1}{n}}\right)\overset{n\to\infty}{\to}\sum_{k=0}^{\infty}\frac{1}{k!} = e$$