ROMANIAN MATHEMATICAL MAGAZINE

**Find:**

$$\Omega = \lim_{n\to\infty}\left(\frac{1}{\left(\frac{n(n+1)}{2}\right)!}\cdot\prod_{k=1}^{n} k!\cdot\binom{\frac{k(k+1)}{2}}{\frac{k(k-1)}{2}}\right)$$

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$$\text{Let: } S_n = \prod_{k=1}^{n} k!\binom{\frac{k(k+1)}{2}}{\frac{k(k-1)}{2}} = \prod_{k=1}^{n} k!\frac{\left(\frac{k(k+1)}{2}\right)!}{\left(\frac{k(k-1)}{2}\right)!k!}$$

$$= \prod_{k=1}^{n}\frac{\left(\frac{k(k+1)}{2}\right)!}{\left(\frac{k(k-1)}{2}\right)!} = \frac{\left(\frac{1\cdot 2}{2}\right)!\left(\frac{2\cdot 3}{2}\right)!\left(\frac{3\cdot 4}{2}\right)!\cdot\ldots\cdot\left(\frac{(n-1)n}{2}\right)!\left(\frac{n(n+1)}{2}\right)!}{\left(\frac{1\cdot 0}{2}\right)!\left(\frac{2\cdot 1}{2}\right)!\left(\frac{3\cdot 2}{2}\right)!\left(\frac{4\cdot 3}{2}\right)!\cdot\ldots\cdot\left(\frac{n(n-1)}{2}\right)!}$$

$$= \left(\frac{n(n+1)}{2}\right)!$$

**Then:**

$$\Omega = \lim_{n\to\infty}\left(\frac{1}{\left(\frac{n(n+1)}{2}\right)!}S_n\right) = \lim_{n\to\infty}\left(\frac{1}{\left(\frac{n(n+1)}{2}\right)!}\cdot\left(\frac{n(n+1)}{2}\right)!\right) = \lim_{n\to\infty}(1) = 1$$

**Therefore:** $\Omega = 1$