ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega=\lim_{n\to\infty}\left(\frac{(2n+1)log\left(\frac{n^n}{n!}\right)}{n^3sin\left(\frac{\pi}{n}\right)}\right)$$

Proposed by Khaled Abd Imouti-Syria

Solution by Daniel Sitaru-Romania

$$\Omega=\lim_{n\to\infty}\left(\frac{(2n+1)log\left(\frac{n^n}{n!}\right)}{n^3sin\left(\frac{\pi}{n}\right)}\right)=\lim_{n\to\infty}\frac{1}{\pi}\cdot\frac{2n+1}{n}\cdot\frac{\frac{\pi}{n}}{sin\left(\frac{\pi}{n}\right)}\cdot\frac{log\left(\frac{n^n}{n!}\right)}{n}=$$

$$=\frac{1}{\pi}\cdot 2\cdot 1\cdot\lim_{n\to\infty}\frac{log\left(\frac{n^n}{n!}\right)}{n}\overset{TCS}{\cong}\frac{2}{\pi}\lim_{n\to\infty}\frac{log\left(\frac{(n+1)^{n+1}}{(n+1)!}\right)-log\left(\frac{n^n}{n!}\right)}{n+1-n}=$$

$$=\frac{2}{\pi}\lim_{n\to\infty}log\left(\frac{(n+1)^{n+1}}{(n+1)!}\cdot\frac{n!}{n^n}\right)=$$

$$=\frac{2}{\pi}\lim_{n\to\infty}log\left(\frac{(n+1)^n}{n^n}\right)=\frac{2}{\pi}\lim_{n\to\infty}log\left(1+\frac{1}{n}\right)^n=\frac{2}{\pi}loge=\frac{2}{\pi}$$