ROMANIAN MATHEMATICAL MAGAZINE

Find:

$$\Omega = \lim_{x\to 0}\frac{(1-2x)^{-\frac{3}{x}}-(1-3x)^{-\frac{2}{x}}}{x}$$

Proposed by Vasile Mircea Popa – Romania

Solution by Pham Duc Nam-Vietnam

$$L=\lim_{x\to 0}\frac{(1-2x)^{-\frac{3}{x}}-(1-3x)^{-\frac{2}{x}}}{x}=\lim_{x\to 0}\frac{e^{-\frac{3}{x}\ln(1-2x)}-e^{-\frac{2}{x}(1-3x)}}{x}=$$

$$=\lim_{x\to 0}e^{-\frac{2}{x}\ln(1-3x)}\lim_{x\to 0}\frac{e^{-\frac{3}{x}\ln(1-2x)+\frac{2}{x}\ln(1-3x)}-1}{x}$$

$$=e^{6}\lim_{x\to 0}\frac{e^{-\frac{3}{x}\ln(1-2x)+\frac{2}{x}\ln(1-3x)}-1}{\left(-\frac{3}{x}\ln(1-2x)+\frac{2}{x}\ln(1-3x)\right)}\frac{\left(-\frac{3}{x}\ln(1-2x)+\frac{2}{x}\ln(1-3x)\right)}{x}=$$

$$=e^{6}\lim_{x\to 0}\frac{2\ln(1-3x)-3\ln(1-2x)}{x^2}$$

Let: $K=\lim_{x\to 0}\frac{2\ln(1-3x)-3\ln(1-2x)}{x^2}, x\to -x$

$$\Rightarrow K=\lim_{x\to 0}\frac{2\ln(1+3x)-3\ln(1+2x)}{x^2}\Rightarrow 2K=\lim_{x\to 0}\frac{2\ln(1-9x^2)-3\ln(1-4x^2)}{x^2}$$

$$=2\cdot 9\lim_{x\to 0}\frac{\ln(1-9x^2)}{9x^2}-3\cdot 4\lim_{x\to 0}\frac{\ln(1-4x^2)}{4x^2}=18+12=-6\Rightarrow K=-3$$

$$\Rightarrow L=-3e^{6}$$

$\because \lim_{x\to 0}\frac{e^x-1}{x}=1, \lim_{x\to 0}\frac{\ln(1-x)}{x}=-1$ are basic limits.