

ROMANIAN MATHEMATICAL MAGAZINE

If $x, y, z \geq -\frac{1}{2}$ such that $x + y + z = -\frac{3}{4}$ and $\lambda \geq \frac{5}{4}$ then find the maximum value

$$P = \sum_{cyc} \frac{2x + 1}{4x^2 + 4x + \lambda}$$

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Since $2x + 1, 2y + 1, 2z + 1 \geq 0$, then we have

$$\begin{aligned} P &= \sum_{cyc} \frac{2x + 1}{(2x + 1)^2 + \frac{1}{4} + \left(\lambda - \frac{5}{4}\right)} \stackrel{AM-GM}{\geq} \sum_{cyc} \frac{2x + 1}{(2x + 1) + \left(\lambda - \frac{5}{4}\right)} = \sum_{cyc} \left(1 - \frac{\lambda - \frac{5}{4}}{2x + \lambda - \frac{1}{4}}\right) \\ &= 3 - \left(\lambda - \frac{5}{4}\right) \sum_{cyc} \frac{1}{2x + \lambda - \frac{1}{4}} \stackrel{CBS}{\geq} 3 - \frac{9\left(\lambda - \frac{5}{4}\right)}{2(x + y + z) + 3\left(\lambda - \frac{1}{4}\right)} = \\ &= 3 - \frac{9\left(\lambda - \frac{5}{4}\right)}{-\frac{3}{2} + 3\left(\lambda - \frac{1}{4}\right)} = \frac{6}{4\lambda - 3}, \end{aligned}$$

so the maximum value of P is $\frac{6}{4\lambda - 3}$, for $x = y = z = -\frac{1}{4}$.