ROMANIAN MATHEMATICAL MAGAZINE

If $x,y,z\geq -\frac{1}{2}$ such that $x+y+z=-\frac{3}{4}$ and

$\lambda\geq\frac{5}{4}$ then find the maximum value of

$$P=\sum_{cyc}\frac{2x+1}{4x^2+4x+\lambda}$$

Proposed by Marin Chirciu-Romania

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

Since $2x+1,2y+1,2z+1\geq 0$, then we have

$$P=\sum_{cyc}\frac{2x+1}{\left[(2x+1)^2+\frac{1}{4}\right]+\left(\lambda-\frac{5}{4}\right)}\overset{AM-GM}{\leq}\sum_{cyc}\frac{2x+1}{(2x+1)+\left(\lambda-\frac{5}{4}\right)}=\sum_{cyc}\left(1-\frac{\lambda-\frac{5}{4}}{2x+\lambda-\frac{1}{4}}\right)$$

$$=3-\left(\lambda-\frac{5}{4}\right)\sum_{cyc}\frac{1}{2x+\lambda-\frac{1}{4}}\overset{CBS}{\leq}3-\frac{9\left(\lambda-\frac{5}{4}\right)}{2(x+y+z)+3\left(\lambda-\frac{1}{4}\right)}=$$

$$=3-\frac{9\left(\lambda-\frac{5}{4}\right)}{-\frac{3}{2}+3\left(\lambda-\frac{1}{4}\right)}=\frac{6}{4\lambda-3},$$

so the maximum value of P is $\frac{6}{4\lambda-3}$, for $x=y=z=-\frac{1}{4}$.