ROMANIAN MATHEMATICAL MAGAZINE

Let $x, y \in \mathbb{R}$ such that $x + y = x^2 + y^2$.

Find the maximum and the minimum value of $A = x^3 + y^3$

Proposed by Nguyen Hung Cuong-Vietnam

Solution by Mohamed Amine Ben Ajiba-Tanger-Morocco

We have $x + y = x^2 + y^2 \geq 0$ **then** $A = x^3 + y^3 = (x+y)\left[\left(x - \frac{y}{2}\right)^2 + \frac{3y^2}{4}\right] \geq 0$,

so the minimum value of A **is** 0, **for** $x = y = 0$.

Now by $\mathrm{AM}-\mathrm{GM}$ **inequality, we have:**

$$4(x+y)^3 A = (x+y)^2.(x+y)^2.4(x^2 - xy + y^2) \leq$$
$$\leq \left(\frac{(x+y)^2 + (x+y)^2 + 4(x^2 - xy + y^2)}{3}\right)^3$$

$= 8(x^2 + y^2)^3$, **then** $A \leq 2$, **so the maximum value of** A **is** 2, **for** $x = y = 1$.