ROMANIAN MATHEMATICAL MAGAZINE

If $a, b, c \in [0,1]$ then find:

$$min\Omega, \qquad max\Omega, \qquad \Omega = a^2 + b^2 + c^2 - abc$$

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Let be $f: [0,1]X[0,1]X[0,1] \to \mathbb{R}, f(x,y,z) = x^2 + y^2 + z^2 - xyz$

$$f''_{xx} = 2 > 0, f''_{yy} = 2 > 0, f''_{zz} = 2 > 0$$

f convex in each variable on $[0,1]X[0,1]X[0,1]$ **−compact.**

By Gireaux's theorem f **−has a minimum and a maximum value in one of the points:**
$(0,0,0), (1,0,0), (0,1,0), (0,0,1), (1,1,0), (1,0,1), (0,1,1), (1,1,1)$

$$f(0,0,0) = 0, f(1,0,0) = f(0,1,0) = f(0,0,1) = 1,$$
$$f(1,1,0) = f(0,1,1) = f(1,0,1) = f(1,1,1) = 2$$

$$min\Omega = 0, max\Omega = 2$$