ROMANIAN MATHEMATICAL MAGAZINE

If $2a^2+b=2b^2+c=2c^2+a=\frac{1}{2}$ then find:

$$\Omega=a+b+c$$

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$$\begin{cases}2a^2+b=\dfrac{1}{2}\\ 2b^2+c=\dfrac{1}{2}\\ 2c^2+a=\dfrac{1}{2}\end{cases}\Rightarrow\begin{cases}4a^2+2b=1\\ 4b^2+2c=1\\ 4c^2+2a=1\end{cases}\Rightarrow\begin{cases}4a^2=1-2b\\ 4b^2=1-2c\\ 4c^2=1-2a\end{cases}\Rightarrow$$

$$a,b,c\in\left(-\infty;\frac{1}{2}\right]$$

$$(*)\begin{cases}4a^2+2b=1\\ 4b^2+2c=1\\ 4c^2+2a=1\end{cases}$$ $\Rightarrow$*Summarize the system of equatons side by side:*

$$(4a^2+2a)+(4b^2+2b)+(4c^2+2c)=3$$

$$\left(2a+\frac{1}{2}\right)^2+\left(2b+\frac{1}{2}\right)^2+\left(2c+\frac{1}{2}\right)^2=\frac{15}{4}\quad(1)$$

$(*)$*The system of equations is symmetric with respect to the variables* a,b,c *.*
Therefore $a=b=c$ *.*

Or from $(*)$ $$\begin{cases}4(a+b)(a-b)=2(c-b)\\ 4(a+c)(a-c)=2(a-b)\\ 4(b+c)(b-c)=2(a-c)\end{cases}\Rightarrow$$

$$64(a+b)(a-b)(a-c)(a+c)(b-c)(b+c)=8(c-b)(a-b)(a-c)$$

$$8(a-b)(b-c)(a-c)(8(a+c)(b+c)(a+c)+1)=0\Rightarrow a=b=c,$$

Then by (1) : $3\left(2a+\frac{1}{2}\right)^2=\frac{15}{4}$

$$a_1=\frac{-1-\sqrt{5}}{4}\ ,\ a_2=\frac{-1+\sqrt{5}}{4}$$

So $a_1=b_1=c_1=\frac{-1-\sqrt{5}}{4}$; $a_2=b_2=c_2=\frac{-1+\sqrt{5}}{4}$

Then $a_1+b_1+c_1=\frac{-3-3\sqrt{5}}{4}$, $a_2+b_2+c_2=\frac{-3+3\sqrt{5}}{4}$